**Application of first order ODE**

**Problem-01:** The population of bacteria in a culture grows at a rate that is proportional to the number at present. Initially, it has number of bacteria, and after 1 hr the number of bacteria is measured to be .

(a). What is the number of bacteria after t hr?

(b). Determine the necessary time for number of bacteria to triple.

**Solution:** Let, be the number of bacteria at any time *t* hr.

Since, the rate of growth of bacteria is proportional to the number of bacteria,

So 



where, k is a proportional constant.

According to the question we have,





Now from Eq. (1) we can write,













From Eq. (2) & Eq.(4) we have,





From Eq. (3) & Eq.(4) we have,









Using the values of *c* & *k* in Eq.(4) we have,



This is the number of bacteria after *t* hr.

Again, let after  hr the number of bacteria will be triple. i.e, .

Now from Eq. (4) we have,











This is the required time. (**ans.**)

**Problem-02:** The population of bacteria in a culture grows at a rate that is proportional to the number at present. Initially, there are 600 bacteria, and after 3 hr there are 10,000 bacteria.

(a). What is the number of bacteria after t hr?

(b). What is the number of bacteria after 5 hr?

(c). When will the number of bacteria reach 24,000?

**Solution:** Let, be the number of bacteria at any time *t* hr.

Since, the rate of growth of bacteria is proportional to the number of bacteria,

So 



where, k is a proportional constant.

According to the question we have,





Now from Eq. (1) we can write,













From Eq. (2) & Eq.(4) we have,





From Eq. (3) & Eq.(4) we have,













Using the values of *c* & *k* in Eq.(4) we have,



This is the number of bacteria after *t* hr.

Again, For  we get from Eq. (5),





This is the number of bacteria after 5 hr.

Again, let after  hr the number of bacteria will be 24,000. i.e, .

Now from Eq. (4) we have,











This is the required time. (**ans.**)

**Problem-03:** Radioactive substances decay at a rate that is proportional to the amount present. The half-life of a substance is the time required for a given amount to be reduced by one-half. The half-life of cesium-137 is 30 years. Suppose we have 100 mg sample.

(a). Find the mass that remains after *t* years.

(b). How much of the sample remains after 100 years?

(c). After how long will only 1 mg remain?

**Solution:** Let, be the amount of cesium-137 present in the sample at any time *t* hr.

Since, the rate of decay of cesium-137 is proportional to the amount,

So 



where, k is a proportional constant.

According to the question we have,





Now from Eq. (1) we can write,













From Eq. (2) & Eq.(4) we have,





From Eq. (3) & Eq.(4) we have,











Using the values of *c* & *k* in Eq.(4) we have,



This is the number of bacteria after *t* hr.

Again, For  years, we get from Eq. (5),





This amount of sample will remain after 100 years.

Again, let after  years the amount of sample will remain 1mg. i.e, .

Now from Eq. (4) we have,











This is the required time. (**ans.**)

**Problem-04:** When a cake is removed from an oven, it’s temperature is measured at  . Three minutes later it’s temperature is . How long will it take for the cake to cool off to a room temperature of ?

(a). Give a relation that gives the temperature of the cake after *t* mins.

(b). How long will it take for the cake to cool off to ?

**Solution:** Let, be the amount of temperature of the cake at any time *t* mins.

From Newton’s Law of Cooling we know that the temperature of a body drops at a rate that is proportional to the difference between the temperature of the body and the temperature of the surrounding medium.

So 



where, k is a proportional constant.

According to the question we have,







Now from Eq. (1) & Eq. (4) we can write,

















From Eq. (2) & Eq.(5) we have,





From Eq. (3) & Eq.(5) we have,











Using the values of *c* & *k* in Eq.(5) we have,



This is the amount of temperature of the cake after *t* mins..

Again, let after  mins. the amount of temperature of the cake will be . i.e, .

Now from Eq. (5) we have,











This is the required time. (**ans.**)

**Exercise:**

**Problem-01:** The population of a certain community increases at a rate that is proportional to the number of people present at any time. If the population has doubled in 30 years, how long will it take to triple?

**Problem-02:** If a small metal bar, whose initial temperature is , is dropped into a container of boiling water. How long will it take for the bar to reach , if it is known that temperature increases in 1second? 